Gradient Based Line Search Techniques for Multi-objective Optimization Problems

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Developing numerical approximation techniques for multi-objective programming problems is a growing research area in recent time. Some of these techniques for unconstrained multi-objective optimization problems include steepest descent method ([7]), Newton method ([6]), quasi-Newton methods ([3,9]) for unconstrained multi-objective optimization problems and SQP method ([8])ans SQCQP method ([5]) for constrained multi-objective optimization problems.

Consider the multi-objective optimization problem

$$(MOP): \min_{x \in X \subseteq \mathbb{R}^n} \quad (f_1(x), f_2(x), ..., f_m(x)),$$

where $f_j : \mathbb{R}^n \to \mathbb{R}$, $j = 1, 2..., m$ are continuously differentiable functions. This seminar topic focuses on some new line search techniques for (MOP), which we have developed during my Ph. D. These methods are free from any kind of priori chosen scalars or ordering information of objective functions as accepted in scalarization methods. The basic structure of these methods rests on the derivation of the descent direction of the optimization problem at every iterating point through subproblems. These subproblems use linear/quadratic approximation of the original problem. A non-differentiable penalty function is used in the constrained multi-objective optimization problems to restrict constraint violations. Convergence of each method is justified under some mild assumptions. Following line search techniques are studied.

- Modified quasi-Newton method for unconstrained multi-objective optimization problems ([3]).
- A globally convergent sequential quadratic programming (SQP) method for nonlinear inequality constrained multi-objective optimization problems ([2]).

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- A sequential quadratically constrained quadratic programming (SQCQP) based method for nonlinear inequality constrained multi-objective optimization problems ([4, 5]).
- Modified SQCQP method for nonlinear inequality constrained multi-objective optimization problems ([1]).

The modified qausi-Newton method is restricted to unconstrained multi-objective optimization problems, whereas other three methods are explored for constrained case. In SQP and SQCQP methods approximated Pareto fronts are developed. Towards the end of the thesis a new technique is developed for selecting initial points to ensure the spreading of Pareto front. All the methodologies are illustrated and compared with existing methods through various test problems. It is observed that these methods provide better result in most of the test problems.

Advantages over existing methods:

- The method of generating such a sequence is free from any kind of priori chosen scalars or ordering information of objective functions like scalarization process.
- Moreover these methods are quite different from heuristic methods due to convergence property.

Key words: multi-objective optimization ; quasi-Newton method; SQP method; SQCQP method; Pareto front

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