Finite Element Methods for Elliptic Optimal Control Problems with Measure Data

Abstract. We analyze both a priori and a posteriori error analysis of finite element method for elliptic optimal control problems with measure data in a bounded convex domain in $\mathbb{R}^d$ ($d = 2$ or 3). The solution of the state equation of such type of problems exhibits low regularity due to the presence of measure data which introduces some difficulties for both theory and numerics of the finite element method. We first prove the existence, uniqueness and regularity of the solution to the optimal control problem. To discretize the control problem we use piecewise linear and continuous finite elements for the approximations of the state and co-state variables whereas piecewise constant functions are used for the control variable. We derive a priori error estimates of order $\mathcal{O}(h^{2-\frac{d}{2}})$ for the state, co-state and control variables in the L^2-norm. Further, global a posteriori upper bounds for the state, co-state and control variables in the L^2-norm are established. Moreover, local lower bounds for the errors in the state and co-state variables, and global lower bound for the error in the control variable are obtained in the case of two space dimension ($d = 2$). Numerical experiments are provided which support our theoretical results.