# Isogeometric Analysis:
# Condition Number Estimates and Multigrid Methods

**Abstract**

In this talk, a brief description of isogeometric methods [1] will be given. The condition number estimates and multigrid solvers for the matrices arising in isogeometric discretization of elliptic partial differential equations will be discussed.

The bounds for the extremal eigenvalues and the spectral condition number of matrices arising in isogeometric discretizations [2] of elliptic partial differential equations in $\Omega \subset \mathbb{R}^d, d = 2, 3$, are presented. For the $h$-refinement, the condition number of the stiffness matrix is bounded above and below by a constant times $h^{-2}$, and the condition number of the mass matrix is uniformly bounded. For the $p$-refinement, it is proved that the condition number is bounded above by $p^{2d+2}4^{pd}$ and $p^{2d}4^{pd}$ for the stiffness matrix and the mass matrix, respectively.

For large problem size, the high condition number of coefficient matrix necessitates the development of fast and robust iterative solvers [3]. The smoothing property of the relaxation method, and the approximation property of the intergrid transfer operators are analyzed for two-grid and multi-grid cycles. It is shown that the convergence of the multigrid solver is independent of the discretization parameter $h$, and that the overall solver is of optimal complexity.

## References

[1] J.A. Cottrell, T.J.R. Hughes and Y. Bazilevs. *Isogeometric Analysis: Toward Integration of CAD and FEA*. Wiley, 2009.

[2] Y. Bazilevs, L. Beirao Da Veiga, J.A. Cottrell, T.J.R. Hughes and G. Sangalli. Isogeometric analysis: Approximation, Stability and error estimates for $h$- refined meshes. *Math. Models Methods Appl. Sci.* **16(7)**, 1031-1090, 2006.

[3] W. Hackbusch. *Iterative Solution of Large Sparse Systems of Equations*. Springer, 1994.