

Talk 1: On locally Lipschitz functions (joint work with Maribel Garrdio, Complutense University, Madrid)

The class of locally Lipschitz functions defined on an arbitrary metric space is uniformly dense in the real-valued continuous functions defined on the space. A properly smaller class of locally Lipschitz functions, the class of Lipschitz in the small functions introduced by Luukkainen, is uniformly dense in the uniformly continuous real-valued functions. We give optimal proofs of these results, the first based on Lipschitz partitions of unity as identified by Frolik, and the second based on a regularization of the initial uniformly continuous function by an appropriate smoothing kernel. Between the continuous functions and the uniformly continuous functions sits the class of functions mapping Cauchy sequences to Cauchy sequences, containing within them those locally Lipschitz functions where balls about each point of a common radius exist on which the function is Lipschitz. While a parallel uniform density result is not valid, we give necessary and sufficient conditions on the domain space for uniform density. We also present necessary and sufficient conditions for pairwise coincide of our three classes of locally Lipschitz functions, and identify the common sets of boundedness for each function class.

Talk 2: Uniform continuity of the product of real functions (joint work with Som Naimpally)

As is well-known, the pointwise product of two bounded real-valued uniformly continuous functions f and g is uniformly continuous, but this condition is hardly necessary (let $f(x) = g(x) = \sqrt{x}$). We produce necessary and sufficient conditions on a pair of uniformly continuous real-valued functions (f, g) defined on an arbitrary metric space so that their pointwise product is uniformly continuous. Our conditions are sufficient for any pair of real-valued functions, and are necessary for a class of pairs properly containing the uniformly continuous pairs. We precisely identify this class and describe it in various ways.